PDHonline Course S164 (4 PDH)

Introduction to Structural Impact

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INTRODUCTION TO IMPACT LOADING

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COURSE CONTENT

INTRODUCTION

At one time or another most engineers run into cases of impact loading. The general problem of impact is extremely complex. A common case of impact—vehicle collision with a traffic barrier—involves large displacements, material non-linearity, elastic and plastic instability, post-buckling strength, coulomb friction and material behavior under high strain rates. Finite element methods can provide an 'exact' solution (in the sense that the modeling assumptions can be tweaked to produce a recognizable match to test results), but reasonable and useful engineering estimates are possible simply from considerations of a few first principles with some simplifying assumptions. The following discussion will illustrate the usual approaches with a discussion of the underlying physics and some examples.

THE ENGINEERING PROBLEM

The physics of impact necessarily involves conservation of energy and momentum. When a moving object strikes a structure the force which decelerates the mass satisfies conservation of momentum. The kinetic energy of the impacting body will be partially converted to strain energy in the target and partly dissipated through friction and local plastic deformation and strain energy 'radiated' away as stress waves. The details are very difficult to predict, but some simple estimates based on first principles can usually result is reasonable estimates for response.

The chief problem usually involves estimation of deformability. The assumption of a rigid impact is generally useless, since rigidity implies an instantaneous velocity change, therefore infinite acceleration and an infinite force. In real structures the deceleration is limited by elastic and plastic deformation, which in effect cushions the blow, and a major 'trick' is making a reasonable estimate the local compliance or stiffness at the point of impact.

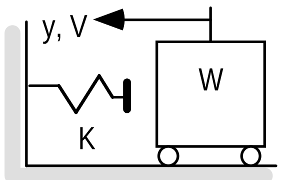
Where impact is a routine service condition, the structure should remain elastic or nearly so and a true dynamic analysis may be required. In many structural or mechanical design problems the requirement is to provide proof that the structure remain substantially intact, even though damaged. Local plastic deformation may be tolerated, provided the overall response is nearly elastic. The St Venant effect allows the local effects to be considered separately as a first approximation. Approaches based on energy equivalence are useful in both elastic and plastic behavior.

Mass and Weight

The difference between mass and weight has been cleverly obscured by generations of academics, textbooks and engineers who don't know better. Many engineers use the terms interchangeably or at best carelessly, which will emphatically not be the case herein. As a working definition, the mass of an object shall mean the quantity of matter making up the object. The mass does not change with location, speed or much else. The weight of an object is the force exerted by gravity acting on the object's mass. Because the mass of an object is usually found by comparing its weight to the weight of a standard, like the International kilogram, force will be considered a basic unit and mass a derived unit according to Newton's second law of motion. (This is an arbitrary choice—ISO measures consider force as the derived unit). Basic units of the example problems are length – inches; force – pounds; time – seconds and mass (= force/acceleration) – lb-sec^2/in. Accordingly an object with a mass of 30 lb-sec^2/in has a weight of F=mg = (30 lb-sec^2/in)(386.09 in/sec^2) = 11582.7 lb. Note how the units cancel because Newton's laws are dimensionally consistent as written–no need for silly artifices like the so-called g_c constant and the use of pounds to refer to both mass and force. Regrettably there is no separate set of examples in ISO units—maybe next time.

ELASTIC RESPONSE

The simplest and most conservative model is to assume that the structure remains perfectly elastic and that the incoming kinetic energy be completely converted to strain energy of deformation within the target structure. The figure shows a simple elastic body, typically a spring with spring rate, k, struck by a mass, m, having a weight, W, moving with a velocity, V.



The impact force, F, carried by the spring and its equal and opposite reaction act to slow the mass and compress the spring a maximum distance y_{max}. The calculation simply equates the work done on the spring to the incoming kinetic energy.

Mass kinetic energy—

$$E_k = \frac{1}{2} mV^2 = \frac{1}{2}\frac{W}{g} V^2$$

Spring energy of deformation—

$$E_p = \int_0^y F du = \int_0^{y_{max}} ku du = \frac{1}{2} k y_{max}^2$$

Equating the kinetic energy to the spring energy, $E_k = E_p$

$$\frac{1}{2} k y_{max}^2 = \frac{1}{2}\frac{W}{g} V^2 \text{ or the equivalent } y_{max} = V\sqrt{\frac{W}{kg}} = \sqrt{\frac{2E_k}{k}}$$

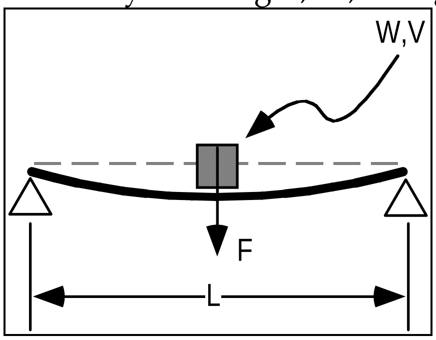
Since the spring force, $F_{max} = k y_{max}$

$$F_{max} = V\sqrt{\frac{kW}{g}} = \sqrt{2kE_k}$$

Although a spring was used to illustrate the process, the actual elastic body could be anything for which the deformation can be estimated. This approach is conservative because it ignores damping, friction and any inelastic deformation or other energy absorption mechanisms. The approach produces reasonable results for assessing such items as bumpers where impact is a service condition and damage is to be prevented and not simply tolerated.

DROPPED MASS

The energy balance approach is easy to extend to impact on a beam by a dropped mass. In this case the energy to be absorbed is the incoming kinetic energy plus the additional work done by the weight, W, acting through the beam deformation



Energy to be absorbed —

$$E_k + Wy_{max} = \frac{1}{2}\frac{W}{g}V^2 + Wy_{max}$$

Equating energy absorbed and the work of elastic deformation—

$$E_k + Wy_{max} = \frac{1}{2}\,ky_{max}^2$$

or

$$y_{max} = \frac{W}{k}\left(1 + \sqrt{1 + \frac{2kE_k}{W^2}}\right) = y_{st}\left(1 + \sqrt{1 + \frac{2E_k}{Wy_{st}}}\right)$$

where y_{st} is the static deflection of the beam under the weight, W. The quantity in parentheses is the dynamic amplification–the factor by which a load is amplified when suddenly imposed. Since the displacement and load are proportional the effective force carried by the beam during impact is the product of the dynamic amplification and the weight, W. Note that the dynamic amplification for V = 0 (E_k = 0) = 2: a load suddenly applied from rest produces twice the stress and twice the displacement as the same load gradually applied.

Remember the assumptions

The beam stiffness is the same for static and dynamic loading.

The beam mass is ignored.

Deformation occurs without energy loss, so in theory the mass rebounds forever. Energy exchanges between kinetic energy of the mass and strain energy of the beam. As you might expect, this assumption is conservative, but it’s frequently sufficient to demonstrate impact resistance.

INELASTIC RESPONSE

In elastic collisions no energy is lost and none goes into yielding or frictional resistance. In practice, designs rugged enough to withstand large collisions elastically are impracticably heavy. Plastic deformation is an excellent and economical means to cushion against impact (as automakers have shown) and the economics often dictates that repair of a structure subject to infrequent damage may be cheaper than producing and using a structure that responds elastically.

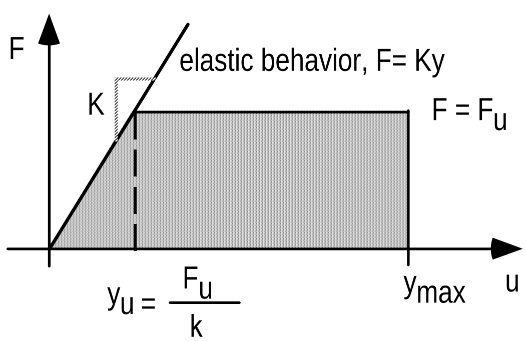
In the elastic idealization the deformation behavior was linear–the force needed to deform the spring or deflect the beam was proportional to the displacement. In practice the force variation is linear only up to the onset of yielding or crushing , which occurs without further increase in resistance. The limiting load may be taken as the load at the onset of yielding or buckling for axially loaded members or the load required to produce plastification of a beam section found according to limit analysis.

The diagram shows idealized elastic plastic behavior in a general way. The shaded area represents the energy absorbed by a structural element undergoing yielding. The displacement increases linearly up to the limit load, F_u after which the structure deforms without additional force. The energy integral is simply the area under the curve

$$E_p = \int_0^y F du = F_u y_{max} - \frac{F_u^2}{2k}$$

Equating the kinetic energy of impact and the strain energy gives the structural deformation, $y_{max} = \dfrac{E_k + \dfrac{F_u^2}{2k}}{F_u}$.

The displacement, y_{max} , is limited in practice by the ability of the structure to absorb plastic deformation without becoming unstable. The ratio of the total deformation to the elastic deformation is conventionally called the 'ductility factor,' μ. The ductility factor relates the elastic capacity of a structure and the impact load in a useful way, using a simple energy balance. Suppose a structure is subject to an external load, F. Equating the work done by the impact force, Fy_{max}, to the energy absorbed by the structure as shown in the loading diagram provides the following energy balance



$$Fy_{max} = \frac{1}{2}F_u y_u + F_u(y_{max} - y_u) \text{ or } \frac{F_u}{F} = \frac{2\mu}{2\mu-1}$$

Where μ = the ductility factor, $\dfrac{y_{max}}{y_u}$.

The relationship expresses the required capacity of the structure for elastic deformation, F_u, as a fraction of the anticipated load and the degree of damage to be tolerated. As a very general rule, ductility factors exceeding 10 are associated with very heavy damage. A ductility factor below 5 produces tolerable damage and will probably allow the structure to continue in use during repair.

IMPULSE AND MOMENTUM

The foregoing makes no explicit reference to Newton's principle of impulse and momentum which also applies. By assumption all the kinetic energy work goes into the static displacement of the spring or beam, and what happens to the impacting mass is ignored. Looking at Newton's Law, $F = ma = m\frac{dv}{dt}$ or $Fdt = mdv$ or by integrating both sides $\int_{\Delta t} Fdt = m \int_{\Delta V} dv = m\Delta V$.

In words impacting objects experience a variable force acting over the time that the two bodies are in contact. The time integral of this force, called the impulse, equals the mass times the change in velocity. It doesn't matter if the behavior is elastic or inelastic, so the impulse-momentum relationship is widely applicable.

Although the impact force varies with time, an average or effective force F_{eff} can be assumed which acts over the impact duration, Δt, such that $F_{eff}\Delta t = \int_{\Delta t} Fdt$. Using physical reasoning to estimate the interval, Δt, then $F_{eff} = \frac{m\Delta V}{\Delta t}$ will provide a reasonable estimate of the impact force.

Many impacts are completely inelastic—the impacting object is simply swept along by a moving target without rebounding, with impacting object or possibly the target being totally or partly crushed. An insect striking an automobile windshield is a common example, but a more important case is a bird strike on an aircraft. In such a case where the impacting object is easily deformable, the forces of deformation will be small with respect to the inertia and the impacting object simply 'squashes.' The impact duration is taken as the time between first contact and the time required for the remainder of the body (gruesomely called the 'bird wedge' in analyses of bird-strike), assumed to continue traveling at the speed of impact, to make contact. Mathematically, duration, Δt, $\approx$ length/impact speed.

Conversely, knowledge of the crushing strength of the target can provide an estimate of the impact duration, using momentum considerations, which in turn can be used with the impact velocity to provide a measure of the penetration distance into such target materials as foamed plastic or wallboard since penetration distance $\approx$ speed x duration.

It's always good practice to make a sanity check of the assumptions and results:

Impacting mass
A soft object may stick or splatter, so not all of the mass decelerates as assumed. An oblique impact may not stop the impacting object completely, so ΔV may not equal the speed just prior to impact

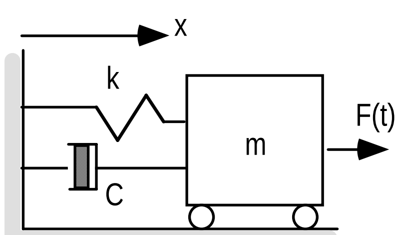
Object geometry
The object may be spinning or tumbling so the length used to figure 'squash-up time' may need adjusting for reasonableness or worst case assumption.

Crushing force magnitude
Some estimate of the crushing force should be made to verify that inertia forces dominate the impact. As shown below, a shorter impact may be less severe in a structurally soft target than a longer impact. Consider the response of the target to the calculated force to check the severity of the impact.

Impact duration
A solid object will stop more quickly than a soft one. The corresponding duration will be smaller and the force larger. The duration estimate for 'squash-up' is more a measure of the time scale of the event and more likely to be an upper bound.

As noted previously, impact forces can rarely be calculated with great accuracy, so any need for refinement or testing is a matter for engineering judgment.

SHOCK RESPONSE



This section will introduce transient force analysis using classical dynamics. The presentation involves some mathematics, but only the minimum needed to handle a few first principles. Rather than discuss a complete solution for transient displacement, the primary consideration will be the dynamic amplification, starting out by considering how the dynamic amplification develops.

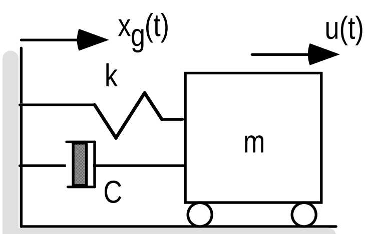
The basic elements of a dynamic system are mass, viscous damping and stiffness, idealized as a particle with mass, m, a linear spring with spring rate, k, and a dashpot with the damping coefficient C. The figure shows how damping is included but for most of this discussion, damping will be ignored. Summing forces on the mass provides the following:

$$\sum F_x = F(t) - kx - Cv = ma \quad \text{or} \quad m\frac{d^2x}{dt^2} + C\frac{dx}{dt} + kx = F(t)$$

The negative signs for the spring and damper forces follow because as shown, the spring and damper both oppose positive displacement. A little algebra to divide through by the mass, m, makes the system handier for engineering use—

$$\frac{d^2x}{dt^2} + 2\zeta\omega\frac{dx}{dt} + \omega^2 x = \frac{F(t)}{m} = \frac{\omega^2 F(t)}{k}$$

The symbols $\omega = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{C}{2\sqrt{km}}$ are the natural frequency (in units of rad/sec = 2π Hz since 2π radians equals one cycle) and critical damping ratio. The natural frequency determines the system stiffness–high frequency systems are considered 'stiff' irrespective of the individual values for k and m. The critical damping ratio defines whether the system is oscillatory. For most structures and machine elements the damping ratio is less than 10% ($\zeta < 0.1$) so they vibrate following a shock. On the other hand automobile suspensions (properly maintained) are highly damped so that striking a pothole does not produce oscillation and the critical damping ratio exceeds 1. Note also that $\frac{F(t)}{k}$ is formally the static deflection and used as the basis for the dynamic amplification.



Forces resulting from base acceleration, $x_g(t)$, rather than applied forces are an important variation in terms of approach. In the case of base motion the development is a little different because the second law of motion requires acceleration to be taken with respect to a non-accelerating reference frame. Taking u as the relative displacement of the mass, m, $x_g(t)$ as the base displacement, so the absolute displacement, $x(t) = x_g(t) + u(t)$. Note that the spring and damper forces depend on the difference between the absolute displacement of the mass, x, and the base movement x_g.

$$m\frac{d^2(x_g + u)}{dt^2} + C\frac{d(x_g + u - x_g)}{dt} + k(x_g + u - x_g) = m\frac{d^2u}{dt^2} + m\frac{d^2x_g}{dt^2} + C\frac{du}{dt} + ku = 0$$

or $$m\frac{d^2u}{dt^2} + C\frac{du}{dt} + ku = -m\frac{d^2x_g}{dt^2} = -ma_g$$

and $$\frac{d^2u}{dt^2} + 2\zeta\omega\frac{du}{dt} + \omega^2 u = -a_g$$

The rephrased relationship in terms of relative motion is then simply the same as that for a time varying force, with the force F(t) taken as the negative of the product of the mass and the base acceleration.

SUDDENLY APPLIED FORCE

A minimum of algebra was promised so the solution for the differential equation for a constant force, suddenly applied, called a stepped load, appears without intervening arithmetic. In fact the solution is easy as shown in all the references, and skeptics may verify that the solution uniquely satisfies the differential equation. The displacement without considering damping is

$$x = \frac{F}{k}(1-\cos\omega t) \qquad x(0) = \frac{dx}{dt}(0) = 0$$

The maximum value of the spring force, kx, is 2F, so the dynamic amplification for a force suddenly applied is 2. The step load is the worst case for shock response except where external kinetic energy needs to be dissipated, such as for a falling mass. Consequently, a factor of two is commonly applied to static loading as a conservative rule of thumb to assess sudden loads, irrespective of the time variation. In the real world the actual load history is seldom known, and often falls into the shadow-zone of mis-handling or abuse, where conservatism is very wise.

If damping is included the solution becomes

$$x = \frac{F}{k}\left[1-e^{-\zeta\omega t}\left(\cos\omega t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega t\right)\right]$$

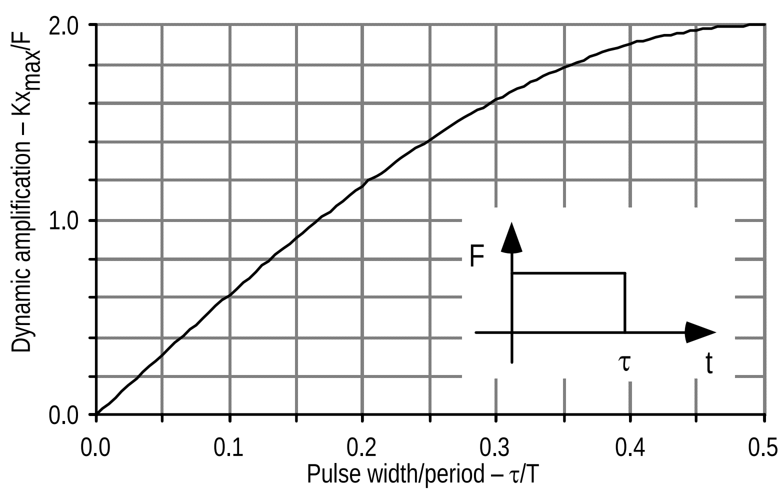
The maximum displacement is $x_{max} = \frac{F}{k}\left(1+ e^{-\zeta\pi}\right)$ so the dynamic amplification is $\left(1+ e^{-\zeta\pi}\right)$, which becomes 2 for the case of zero damping as expected. In practice damping is rarely included in calculations of this sort since the damping ratio or the applied load are seldom known with any accuracy, the effect of damping is small, and hair-splitting in this area has little practical value.

RECTANGULAR PULSE

Intuitively the response to a pulse should depend on the pulse duration, τ, and the system natural frequency, ω. The maximum response to a long step occurs at the instant when $(1-\cos\omega t) = 2$ for which $\omega t = \pi$. This is half the period of vibration of the system, $T = \frac{2\pi}{\omega}$. Consequently rectangular pulses longer than half the system period produce the same dynamic amplification as the stepped force. Conversely pulses much shorter than half the system period don’t act long enough to produce significant displacement, so the amplification is less than 2. In fact the maximum displacement for pulse widths, τ less than half the system period is

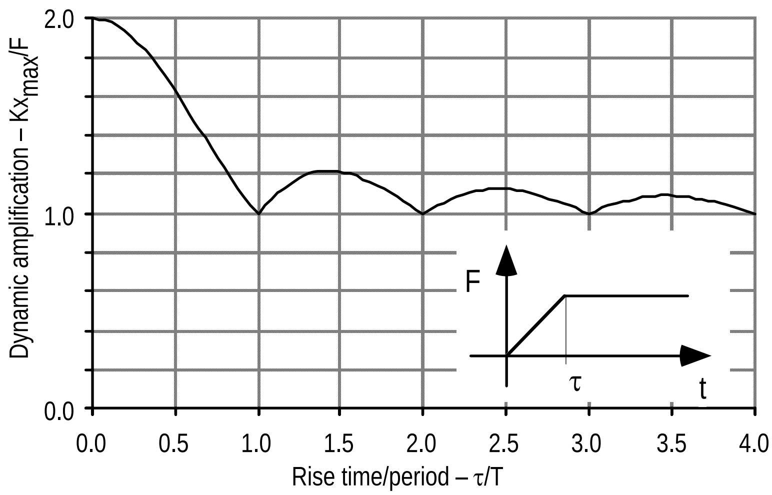
$$x_{max} = \frac{2F}{k}\sin\left(\frac{\pi\tau}{T}\right) \qquad \text{valid for } \tau \le \frac{T}{2}$$

For skeptics, Reference 1 has a simple, elegant proof of that relationship unobscured by excess arithmetic. The figure shows the complete dynamic amplification for the rectangular pulse, which in this case varies with the ratio of pulse width to system period as we anticipated. The figure shows that softer systems amplify shock loads less than stiffer systems, and in fact the pulse response level is a way to distinguish a stiff system from a soft one. This variation gives a rational means for assessing the severity of a shock load. Reference 2 shows that most pulse or spike loadings show this same sort of variation with peak dynamic amplification occurring for pulse widths between 50% and 100% of the system period.



RAMPED STEP

A final variation on the stepped force response is the ramped step shown in the figure. Again, the figure shows what we know intuitively, that the more gradual the application of force the less severe the amplification. The maximum displacement is



$$x_{max} = \frac{F}{k}\left(1 + \left|\frac{T}{\pi\tau}\sin\left(\frac{\pi\tau}{T}\right)\right|\right)$$

The dynamic amplification for a very short rise time, τ, equals 2 because the force transient is more nearly a stepped force. The more gradual the rise–the longer the rise time–the lower the dynamic amplification, because the load is applied more nearly statically. The diagram can also be used to distinguish a static load from a dynamic load.

REFERENCES

The following references contain details of much of what is presented above in summary. Reference 1 is particularly recommended as a good practical introduction to the subject. Reference 2 is a handbook with much valuable supporting information. References 3 and 4 are standard textbooks which are as good as any and better than most.

1. Irvine, H. M., *Structural Dynamics for the Practising Engineer*, Allen and Unwin, London, 1986.

2. Harris, Cyril and Crede, Charles, *Shock and Vibration Handbook*, McGraw-Hill, New York, 1976.

3. Timoshenko, S. P., Young, D.H., Weaver, W., *Vibration Problems in Engineering*, Wiley, New York, 1974.

4. Den Hartog, J. P., *Mechanical Vibrations*, Dover Publications, New York.