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LECTURE

17

Chapter 9.5 – 9.6

BEAMS: DEFORMATION BY SINGULARITY FUNCTIONS

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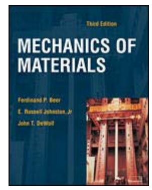

by

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ENES 220 – Mechanics of Materials

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Singularity Functions

- Introduction
  - The integration method discussed earlier becomes tedious and time-consuming when several intervals and several sets of matching conditions are needed.
  - We noticed from solving deflection problems by the integration method that the shear and moment could only rarely described by a single analytical function.

Singularity Functions

- Introduction
  – For example the cantilever beam of Figure 9a is a special case where the shear *V* and bending moment *M* can be represented by a single analytical function, that is

$$V(x) = w(L - x) \qquad (15a)$$

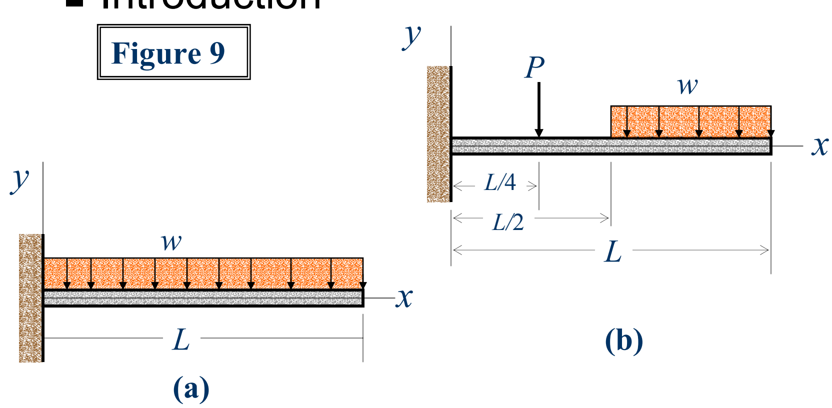
and

$$M(x) = w\left(-L^2 + 2Lx - x^2\right) \qquad (15b)$$

Singularity Functions

- Introduction

Figure 9



Singularity Functions

- Introduction
  - While for the beam of Figure 9b, the shear *V* or moment *M* cannot be expressed in a single analytical function. In fact, they should be represented for the three intervals, namely

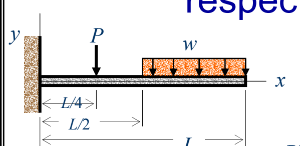
    $0 \le x \le L/4$,

    $L/4 \le x \le L/2$, and

    $L/2 \le x \le L$

Singularity Functions

- Introduction
  - For the three intervals, the shear *V* and the bending moment *M* can are given, respectively, by

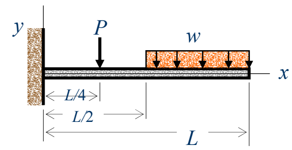


$$V(x) = \begin{cases} P + \dfrac{wL}{2} & \text{for } 0 \le x \le L/4 \\ \dfrac{wL}{2} & \text{for } L/4 \le x \le L/2 \\ \dfrac{wL}{2} - w\left(x - \dfrac{L}{2}\right) & \text{for } L/2 \le x \le L \end{cases}$$

Singularity Functions

- Introduction

  and



$$M(x)=\begin{cases} -\dfrac{PL}{4}-\dfrac{3wL^2}{8}+Px+\dfrac{wL}{2}x & \text{for } 0\le x\le L/4 \\ -\dfrac{3wL^2}{8}+\dfrac{wL}{2}x & \text{for } L/4\le x\le L/2 \\ -\dfrac{3wL^2}{8}+\dfrac{wL}{2}x-\dfrac{w}{2}\left(x-\dfrac{L}{2}\right)^2 & \text{for } L/2\le x\le L \end{cases}$$

Singularity Functions

- Introduction
  - We see that even with a cantilever beam subjected to two simple loads, the expressions for the shear and bending moment become complex and more involved.
  - *Singularity functions* can help reduce this labor by making *V* or *M* represented by a single analytical function for the entire length of the beam.

Singularity Functions

- Basis for Singularity Functions
  - Singularity functions are closely related to he unit step function used to analyze the transient response of electrical circuits.
  - They will be used herein for writing one bending moment equation (expression) that applies in all intervals along the beam, thus eliminating the need for matching equations, and reduce the work involved.

Singularity Functions

- Definition

A singularity function is an expression for *x* written as $\langle x - x_0 \rangle^n$, where *n* is any integer (positive or negative) including zero, and x_0 is a constant equal to the value of *x* at the initial boundary of a specific interval along the beam.

Singularity Functions

- Properties of Singularity Functions
  - By definition, for $n \geq 0$,

$$\langle x - x_0 \rangle^n = \begin{cases} (x - x_0)^n & \text{when } x \geq x_0 \\ 0 & \text{when } x < x_0 \end{cases} \quad (16)$$

  - Selected properties of singularity functions that are useful and required for beam-deflection problems are listed in the next slides for emphasis and ready reference.

Singularity Functions

- Selected Properties

$$\langle x - x_0 \rangle^n = \begin{cases} (x - x_0)^n & \text{when } n > 0 \text{ and } x \geq x_0 \\ 0 & \text{when } n > 0 \text{ and } x < x_0 \end{cases} \quad (17)$$

$$\langle x - x_0 \rangle^0 = \begin{cases} 1 & \text{when } n > 0 \text{ and } x \geq x_0 \\ 0 & \text{when } n > 0 \text{ and } x < x_0 \end{cases} \quad (18)$$

Singularity Functions

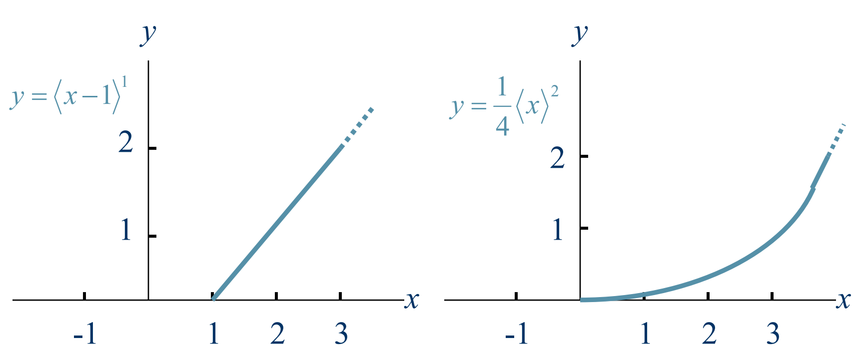
- Integration and Differentiation of Singularity Functions

$$\int \langle x - x_0 \rangle^n dx = \frac{1}{n+1} \langle x - x_0 \rangle^{n+1} + \mathrm{C} \quad \text{when } n > 0 \qquad (19)$$

$$\frac{d}{dx} \langle x - x_0 \rangle^n = n \langle x - x_0 \rangle^{n-1} \quad \text{when } n > 0 \qquad (20)$$

Singularity Functions

- Examples: Singularity Functions



(a) **Figure 10** (b)